• Let V be a finite dimensional vector space, and let $T : V \to V$ be a linear mapping. T is said to be *diagonalizable* if there exists a basis of V, all of whose vectors are eigenvectors of V.

• $T : V \to V$ is diagonalizable if and only if, for any basis α of V, the matrix $[T]_\alpha^\alpha$ is similar to a diagonal matrix.

• In order for a liner mapping or a matrix to be diagonalizable, it must have enough *linearly independent eigenvectors* to form a basis of V.

• Let $\mathbf{x}_i$ $(1 \leq i \leq k)$ be eigenvectors of a linear mapping $T : V \to V$ corresponding to distinct eigenvalues λ_i. Then $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k\}$ is a linearly independent subset V.

• For each $i\,(1 \leq i \leq k)$, let $\{\mathbf{x}_{i,1}, \mathbf{x}_{i,2}, \ldots, \mathbf{x}_{i,n_i}\}$ be a linearly set of eigenvectors of T all with eigenvalue λ_i and suppose the λ_i are distinct. Then

$$S = \{\mathbf{x}_{1,1}, \ldots, \mathbf{x}_{1,n_1}\} \cup \{\mathbf{x}_{k,1}, \ldots, \mathbf{x}_{k,n_k}\}$$

is linearly independent.

• Let V be finite-dimensional, and let $T : V \to V$ be linear. Let λ be an eigenvalue of T, and assume that λ is an m-fold root (a root of multiplicity m) of the characteristic polynomial of T. Then we have

$$1 \leq \dim(E_\lambda) \leq m.$$

• Let $T : V \to V$ be a linear mapping on a finite-dimensional vector space V, and let $\lambda_1, \lambda_2, \ldots, \lambda_k$ be its distinct eigenvalues. Let m_i be the multiplicity of λ_i as a root of the characteristic polynomial of T. Then T is diagonalizable if and only if

(1) $m_1 + m_2 \cdots + m_k = n = \dim(V)$, and

(2) for each i, $\dim(E_{\lambda_i}) = m_i$.

• Let $T : V \to V$ be a linear mapping on a finite-dimensional space V, and assume that T has $n = \dim(V)$ distinct real eigenvalues. Then T is diagonalizable.